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CONTRADICTIONS BETWEEN THE THEORY OF CHOICE AND PRACTICAL SOLUTIONS OF ECONOMIC DECISION MAKING

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ABSTRACT
The theory of choice between variants is one of the most important fields of economics mainly because of the future implications of such decisions. The paper addresses three particular contradictions in the development of the theory. The first appears between Daniel Bernoulli and von Neumann–Morgenstern. While the former considers utility as the logarithm function of money wealth, the latter identifies it with the sum of money wealth. The second contradiction is related to the treatment of psychological factors, which arises in the use of objective and subjective probability. The third contradiction is a distinction between ordinal and cardinal utility, which used to be strong, but has become less relevant recently. The train of thought reveals the main nodes of theoretical development.

JEL codes: C20, B40, D90, M20, G31, D92, D81

Keywords: rational choice, ordinal and cardinal utility, psychological factor, decision theory

1 INTRODUCTION

> *"Alternative descriptions of a decision problem often give rise to different preferences, contrary to the principle of invariance that underlies the rational theory of choice. Violations of this theory are traced to the rules that govern the framing of decision and to the psychophysical principles of evaluation embodied in prospect theory. Invariance and dominance are obeyed when their application is transparent and often violated in other situations. Because these rules are normatively adequate but descriptively inaccurate, there is no theory of choice that is normatively adequate and descriptively accurate."*
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> (Kahneman–Tversky, 1986:S251).

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Theorists of economic decision-making have been searching to find the criterion being the foundation of choice between modes of action (variants) for over a century. There have been repeated efforts to present adequate answers on why the normative / descriptive models of the theory of choice lack consistency. The quest in theory and practice oscillated widely, which had several reasons. The inability to accurately measure utility has been the biggest problem all along. Another major issue has been related to the consideration of the metaphysical notion, i.e., how to treat mental-psychological aspects. In the course of practical path finding, the part played by ordinal versus cardinal utility, subjective versus objective application of probability and value versus wealth in the theory of choice have become significantly relativised. Note that many of the most influential thinkers of economics including Keynes, Hicks and Samuelson to some extent, did not consider utility to be the foundation of the paradigm of the theory of choice. By the mid-20th century, the maximisation of expected subjective utility had been in the focus of the theory of decision-making in economics. A pragmatic approach had become paramount, i.e., to define which model of the methodology could be best used to make effective decisions. The theorists of decision-making had to face a challenge, namely, having to choose between theoretical adequacy and operational practicability.

The theory of utility is a major concept in economics. It explains the behaviour of individuals in making decisions by assuming they are capable – in a consistent manner – to rank their choices based on their preferences. According to the utility theory, individuals have their own utility functions and act in line with them. The utility theory by the dominant neo-classical paradigm relies on the idea that individuals attach an imaginary utility value to initial monetary value when making decisions. Decision makers take the different levels of monetary values, then transform them into different hypothetical values (utilises) and make their decisions based on those utility values (rather than economic values) while the result is re-transformed into monetary values. Since utility signals satisfaction, the individuals' decision-making behaviour is aimed at maximising utility rather than maximising wealth. The two organic models of utility maximisation can be presented as follows:

Daniel Bernoulli (1738/1954) applied the utility function $U(W)= lnW$ to represent the utility offered to individuals by W wealth.

Von Neumann–Morgenstern (1944) provided an axiomatic proof of maximising expected utility, where the utility function is $E(U) = \sum_{i=1}^{n} p_i U_i$.

Moscati (2023) states that the expected utility models by Daniel Bernoulli and von Neumann–Morgenstern relate to decision making *at risk,* where the objective probabilities of alternative events are available, and decision makers are aware of them. However, many decisions are made under the terms of uncertainty, where

objective probabilities are either not available or are unknown to the decision makers.

Savage (1954:97) relied on von Neumann–Morgenstern's utility management and used Ramsey's (1931) and de Finetti's (1931) idea of subjective probability to build his theory of expected subjective utility. Savage's model comprises two elementary concepts: on the one hand, outcomes (and states), and on the other hand, a list of possible scenarios. The outcome connects the result of individual choice with the manifestation of the states. Savage believes when individuals opt for an action, they will not use numerical data, utility or probabilities for justification, but they will rely on simple frameworks containing the description of probable states, and the specific outcomes originating from the mode of action chosen in each state from among the multitude of modes of action.

Gilboa (2009:94–112) demonstrated that both the function of utility and the measure of probability exist because individuals maximise their utility expectations based on the measure of probability during their decisions.

Based on the latter statements, the theory of expected subjective utility synthetised by Savage (1954) had its historical origins in two different traditions. One was the tradition of the theory of choice by Daniel Bernoulli–von Neumann–Morgenstern, and the other was that of mathematical philosophy of subjective probability that can probably be traced back to Bayes, and which was revived in works by Ramsey and de Finetti.

Moscati (2016; 2018) underlined the importance of a heated debate on von Neumann–Morgenstern's model of expected utility from the mid-1940s to the mid-1950s, during which the exact content of the axioms of the model were clarified as well as the question if it was adequate descriptively and/or normatively.

Tests to validate the descriptive character of expected utility commenced at the beginning of the 1950s with pioneering steps headed by Mosteller–Nogee (1951). One can say of the results, just as of the results of tests lasting till the middle of the 1960s, that most thinkers of the decision theory interpreted them as supporting the descriptive validity of the theory of expected utility.

An earlier experiment by Allais (1953) and a thought experiment by Ellsberg (1961) called attention to patterns of choice violating the principle of expected utility. Nevertheless, the doubts of Allais and Ellsberg regarding the descriptive validity of the model had been neglected until the mid-1960s, because they had been thought to be specific cases. Moscati (2016) emphasised economists of the theory of choice were trying to mitigate the violations of the principle of expected utility. They either regarded the patterns of choice presented by Allais and Ellsberg as errors to be responded to with decision makers' corrections or said they had been linked to fictitious situations of choice that rarely occurred in real life. However, a

shift could be observed in the middle of the 1960s. A series of tests proved the patterns of choice presented by Allais and Ellsberg were, in fact, frequent and could be predicted. Decision theorists no longer regarded them to be errors and did not recommend their correction (more details on the tests can be found in Schoemaker, 1982; Moscati, 2018). The above experimental evidence undermined the earlier confidence in the validity of the descriptive nature of expected utility and urged efforts to find alternative models of choice at risk and under uncertainty, which actually commenced in the middle of the 1970s.

Kahneman–Tversky's (1979) theory was a major development in the history of the theory of choice. According to the creators of this theory of psychological origins, the principle of expected utility in economics, as a positive theory, systematically differs from the real world of individual decision making. Heukelom (2007) believes the new theory is a criticism of the principle of expected utility mostly because the followers of the latter *do not pay attention to the descriptive realism of a theory and are only interested in the predictive worthiness of their theory.* With their test, Kahneman and Tversky took a step forward to rationalise the decision-making behaviour of individuals in the real world. What they termed prospect theory explicitly "turned the clock back" to the time preceding von Neumann–Morgenstern (1944). In the prospect theory utility is not an absolute given although it is subject to the individuals' subjective perception. Heukelom (2007) also underlines that, in the studies by Kahneman, Tversky and their followers the psycho-physical differentiation between normative (objective) and descriptive (subjective) approach has become more manifest. Individuals' perceptions of subjective utility and probability differ from their objective values, while a psychological train of thought focuses on a conceptual definition and measurement of subjective utilities and probabilities.

Judged impartially, one should say the merits of the expected utility model by von Neumann–Morgenstern have not been overshadowed by the test results indicating that maximisation of expected utility are not suitable to predict decision makers' behaviour. Schoemaker (1982) believes one cannot say the theory of expected utility can be neglected. On its own, the model provides a deep insight into and a subtle approach to decision making at risk both in terms of its descriptive character and normativity. According to Schoemaker, it reveals that individuals perceive and solve problems differently and offer a framework and language to analyse the differences. While acknowledging the intellectual debt of decision theory to the model of expected utility, Schoemaker voices criticism regarding the dominant paradigmatic status of the model. Nevertheless, *maximising expected utility will remain a valuable benchmark until a richer model of rational choice appears.* Decision makers' behaviour can and should relate to it.

Gilboa–Marinacci (2013) states none of the new models offering an alternative to the theory of expected utility are accepted in such consensus as the theory of expected utility. A survey by the authors have found:

> *"It is not clear if decision making under risk and uncertainty will replace the expected utility model and whether a single paradigm will emerge, it is probably too early to say that it will be the one (like prospect theory)"* (Gilboa-Marinacci, 2013:232).

Moscati (2023) notes people offering alternative decision models instead of the principle of expected utility often criticise the approach to decision modelling in economics adopted by Friedman (1953), Savage (1954) and other mainstream thinkers, because the latter usually defend the principle of expected utility against the suspicion it lacks psychological reality.

The quest for a principle of choice among variants in economic decision theory has been a process fraught with contradictions; it had promising forward momentums and relapses often difficult to understand, it had relativisation and compromises based on pragmatic considerations. Let us look at it from the beginning.

2 PREHISTORY OF UTILITY-BASED RATIONAL CHOICE

The solution of expected utility by Daniel Bernoulli (1738) – where utility rather than monetary wealth was rendered the criterion of rational choice – was the moment when the theory of expected utility was born. Daniel Bernoulli (1738) (hereinafter: DB).[2]

The general problem DB faced was how to identify the reasonable price of the game. Vivian (2013) compared that approach to Huygens's (1657/1920) view that considered the expected value to be the reasonable price of the game. DB, however, suggested they should write *benefit* (emolumentum in Latin) originating from money instead of money. So, the benefit to be achieved from the game had become its reasonable price. In the English translation (1954), the Latin for "benefit" changed into "utility" while the term „emolumentum" was modified to mean *moral expectation*. DB wrote about it as follows:

> *"By multiplying each expected benefit by the number of cases in which it can occur and dividing the sum of these products by the total number of possible cases, an average benefit (emolumentum medium) will be obtained. The gain*

2 Daniel Bernoulli published his work in 1738 in Latin entitled *"Specimen Theoria Nova de Mensura Sortis (Exposition of a New Theory on the Measurement of Risk)"*.

> *corresponding to this benefit will be equal to the value of the risk alternative in question"* (op. cit. 24, D. Bernoulli, 1738/1954:24).

Moscati (2023) emphasises DB presented his theory as "new"[3] in sharp contrast to the dominant theory of the time, the principle of expected pay-off. According to DB, the disadvantage of the principle of expected pay-off is that it eliminates all personal elements from the evaluation of risky variants.

> *"It is clear that all men cannot apply the same rule to measure risk and that, the rule – the expected pay-off principle – should be rejected."* (D. Bernoulli, 1738/1954:24).

According to the traditional outcome of the St. Petersburg game, the expected value of the game is infinite. Todhunter (1865:220) calls attention to a peculiar paradox:

> *"(…) it is accepted no prudent (…) man would be willing to pay even a small number of shillings (dollars) to play St. Petersburg game"* (Todhunter, 1865:220).

According to the traditional "entrepreneurial" solution of the St. Petersburg game, the player may be willing to pay high sums for the game, however, a cautious player would only pay a limited amount for it. This is based on the paradox explained by Todhunter:

> *"The paradox then is that the mathematical result apparently and directly confronts with what common sense dictates"* (1865:220).

Thus, one can say *the direction of mathematical and behavioural observation is divergent.* Searching for a solution of the St. Petersburg paradox, Daniel Bernoulli accepted the traditional outcome of „expected monetary value = ∞", but he turned his attention from the maths of the game to *rational behaviour* and explained the behaviour of players who were only willing to pay a modest sum for the game.

DB provided the solution of the St. Petersburg game, and his train of thought was simple: one should not use the objective value of the game but its merit, i.e., utility for assessment. He summed up his arguments as follows:

> *"The value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal*

3 Moscati (2023) calls attention that Daniel Bernoulli (1738) reproduced in the final part of his study the letter sent by Cramer to Nicolas Bernoulli in 1728-ban, which was forwarded to Daniel in 1732 by Nicolas. Daniel acknowledges Cramer has proposed a theory identical to his one preceding him. At the same time, Daniel Bernoulli states he had presented his hypothesis at the Academy of St. Petersburg before learning about Cramer's theory (Daniel Bernoulli's work was *Papers of the Imperial Academy of the Sciences in Petersburg* [1738]). Daniel Bernoulli ads, Cramer's and his approaches are close to each other, and it seems to him: "*it's amazing we have come to the same result independent of each other"* (Daniel Bernoulli, 1738/1954:33).

> *for everyone; its utility, however, is dependent on the particular circumstances of the person making the estimate. Thus, there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount"* (D. Bernoulli, op. cit. 24).

DB defines utility as a marginally decreasing function of wealth, where wealth is the sum of an individual's total wealth and its money-yielding capacity. Thus, the utility from the given monetary gain will be decreasing as the initial wealth increases. According to DB, the relationship can be described with the natural logarithm function[4]. DB says the players do not make a linear analysis of potential gains, but they assess the marginal gains arising on the basis of a concept termed "moral expectation". So, instead of multiplying probabilities with linear gains, probabilities should be multiplied with the moral expectation of marginal wealth.[5]

In his recent analysis of DB's interpretation, Moscati (2023) linked individuals' risk attitudes to utility using DB's interpretation for a starting point. Thus, DB states what individuals prefer or should prefer under risk conditions is because preference operates under the conditions of certainty. Another key element of DB's theory is a special factor linked to risk-free money which explains how an individual relates to risk. In fact, the above theoretical approach leaves no room for other – for instance, psychological – factors, particularly to ones linked to risk situations, which could be added to the utility of money to make an individual's risk attitude explainable, which is obviously considered in DB's theory. In his theory, the approach to risk is not specific, it can rather be regarded to be a simple psychological phenomenon of an individual. It is fully defined by the individual's attitude to risk-free money, particularly, because DB assumes a concave utility function, i.e., the marginal utility of money is decreasing for him, i.e., each individual is or should be risk averse in the sense that they reject or should reject an actuarily reasonable game.[6]

4 As Moscati (2023) indicates, Pareto (1896) agreed with Daniel Bernoulli's theory, however, he stated there is no explicit reason to assume the logarithmic form of the function of money utility; a square root function or another concave function can also be selected.

5 Moscati (2023) underlines that Daniel Bernoulli focused on objective probabilities decision makers were probably aware of. Nicolas Bernoulli – Daniel Bernoulli's cousin – proposed decision makers should attach subjective weights to some objective probabilities, namely, zero subjective weight to low-value objective probabilities. Daniel Bernoulli rejected the proposal.

6 Daniel Bernoulli's idea was the utility theory could be built in the centre of economics; however, it materialised through Adam Smith's value to utility relation. Adam Smith (1776) called attention to the fact there is a difference between exchange value (price) and utility value (use, usefulness). He illustrated this with the example of diamonds and water. Diamonds are very expensive with

DB's theory was studied by philosophers and mathematicians in the 18th and 19the centuries. Laplace (1812:432–445) referred to it as an expectation. His work is characterised by the differentiation of physical wealth (material benefit) and moral wealth (moral benefit). Utility as a theory was only included in economics much later (in the 1870s), when it seemed the price of goods was subject to their utility, which is a benefit for the individual in the economy. Launching a philosophical-economic thought immediately preceding the above, Bentham (1789) believed the goal of human action is to seek pleasure and to avoid pain. All goals or actions should be considered from the aspect of whether they cause pleasure or pain. Such characteristics are called the utility of an object: *pleasure appears as positive utility while pain is negative utility.* Thus, the goal of actions is to achieve maximum utility. On the basis of the above, Heukelom (2007) could state that such *hedonism* relating to the future can easily be transformed into a theory of choice. Individuals will choose the alternative from the multitude of available ones the one that offers the highest surplus of positive utility with the difference of positive utility above negative utility. That concept of utility maximisation is the core of the utility theory of choice.

DB' theory seamlessly fitted into the framework of marginalist economics. Jevons (1878) and other thinkers of utility re-interpreted DB's concept of benefit/satisfaction (emolumentum) and Cramer's moral value as something equivalent to the concept of utility. Moscati (2013) demonstrated Jevons and other marginalists were not interested in ranking utility differences, instead, early marginalists sought to measure utility, which – for them – represented the expression of the utility of an item of goods as the multiple of the utility of another item of goods taken as the unit measure. Jevons (1879:173–174) did not only relate DB's hypothesis to the game but also to trade. He wrote the following:

> *The basic structure of human habituation is dependent on "uncertain events" from a choice between nodes – as in normal life – from the sequence of a set of emotions associated with future events multiplied by the fraction denoting probability.*

In a comprehensive and in-depth analysis about the formation of rational choice and the economics of utility-maximisation, Bruni (2010) revealed the utilitarian psychological roots of early neo-classical economics. The creators of the set of concepts presumed there exists a one-dimensional, interpersonal comparative measure of mental states (pleasure). The hypothesis that rational individual

a high value of exchange, while they are less useful, but water is cheaper while its utility value is high (cited by Vivian, 2013).

choice means maximising that measure was also there. The fact that the measure had never been found might have caused some disturbance though.

At the end of the 19th century mainstream methodology regarded economics to be a deductive discipline built on a priori laws deduced from self-observation. *Psychological self-observation* appeared to be attained via carefully expressed laws governing individuals' behaviour (such as, a strive for riches). An abstract, positive and deductive economics was construed on the laws.

One needs some time to recognise the roots of utilitarian psychology, which was the foundation of early neo-classical economics. Both Fechner (1860) and Weber (1846) defended the thesis of the measurability of mental phenomena. Fechner voted for the possibility of empirically measuring perception; his relevant work was published on the eve of the marginalist revolution. In his work gaining recognition at the time of the marginalist revolution, Wundt (1873–1874) categorised empirical perceptions by their intensity, durability and modality.[7] Next, Wundt established a three-dimensional system of perceptions (joy-joylessness, tension-relaxation, excitement-depression), however, he had no followers among economists who had founded their theory on one-dimensional utility (pleasure-pain).

Bruni (2010) recalled Edgeworth (1881) had made a peculiar comment on the part played by mental factors. He was of the opinion that ideas by Fechner, Weber and Wundt as well as the results of experimental psychology were important to provide theoretical proof for measuring pleasure in the utility theory. In setting up his axiom, Edgeworth (1887) stated:

> *"Pleasure is measurable, and all pleasure are commensurable"* (Edgeworth, 1881:59).

He also added:

> *"Wundt has shown that sensuous pleasures may thereby be measured, and as utilitarians hold, all pleasure are commensurable…and as the growth rate of pleasure decreases its average increases* (op. cit. 60–61).

As a marginalist, Edgeworth attached great importance to the achievements of experimental psychology particularly to *psychophysics.* In one of his works, Edgeworth (1877) made efforts to establish economics on the grounds of psychology, particularly, on psychophysics developed by Weber, Fechner and Wundt. The the-

7 Wundt's theses are connected to Jevons's (1879) and Edgeworth's (1881) theory of usefulness and similarities can also be found with Bentham's characteristics of pain and pleasure, such as durability or substitutability.

ses of psychophysics represented the path to be followed by Edgeworth both with reference to hedonism and its being founded on experimental facts.[8]

At the beginning of the 20th century, DB's theory met with increasing scepticism. Both Knight (1921) and Keynes (1921) expressed doubts whether numerical probabilities can fully express the way individuals perceive uncertain options, therefore they questioned the idea that expected utility based on numerical probabilities could explain decisions made under the conditions of uncertainty. Hicks (1931) and others said individuals would rather assess their options on the basis of other elements of uncertain pay-off-distribution than the expected utility of payoffs.

The next problem connected to expected utility, the so termed ordinal juncture in decision theory, appeared in the first decades of the 20th century. As seen above, cardinal utility had been an important concept of utility in early neo-classical economics. Bentham (1789) regarded utility as the result of measuring pleasure based on self-observation with its roots in metaphysics. In the case of ordinal utility, a decision-maker simply ranks choices by their preferences without attaching any exact numerical value to utility.

3 BIRTH OF THE RATIONAL THEORY AND THE ORDINAL JUNCTURE

At the beginning of the 20th century, decision theorists dismissed utilitarian assumptions but retained the bigger part of the construct built on them. As Bruni (2010) underlines: the foundations of the theory had changed, axioms and preferences had become the new bases, or they spoke about choices. Rationality had been construed as the consistence of preferences and choices unlike the earlier instrumental rationality of seeking pleasure.

Pareto's (1900) work was the first to lay the grounds for the modern rational theory of choice. Pareto broke with the marginalist-hedonist line of theory in favour of two new goals: he replaced cardinal utility with the ordinal utility function while he established the preference index on the bare facts of choice. Pareto presented his theory of choice at the turn of the 19th and 20th centuries after developing his *action theory* of "logical-non-logical". It should be noted that his action theory had earlier been based on pleasure.

8 Fechner's (1860) law was the basis to measure stimulation and utility, which had little impact on economics. Edgeworth applied the law to measure pleasure and usefulness. (Cf. Bruni, 2010).

In his analysis, Bruni (2010) places special emphasis on the significance of Pareto's action theory supporting it with Pareto quotes. Real action, by Pareto, includes two main components:

- *"the logical component, based on the pure instrumental reasoning, where the means are adequate to the end, subjectively and objectively;*
- *the non-logical one, where non-logical does not mean illogical or irrational but just based on a different type of logic, as Pareto many times has specified"* (Pareto, 1900:162).

According to Pareto, economics simply analyses a *small slice* of human behaviour and action, nevertheless, aspects outside logic should also be given attention:

> *"born of non-logical actions, which also need to be studied. Here difficulties begin to arise"* (op. cit. 162).

Bruni (2010) believes Pareto's theory of choice is more complex than it was believed by his interpreters. According to Pareto, to understand human actions and social operation, logical and non-logical aspects must first be analysed separately and then synthetised. Human actions cannot be understood without synthesis either in the economy (with the dominance of the logical aspect) or in areas outside it. If you want to understand social operations – since, according to Pareto, economics deals with a narrow slice of individual and social life – their logical and non-logical aspects must be studied separately, as the two parts are based on completely different rational variants that cannot be reduced to a single genre.

Pareto had a completely different view on the idea of action than the mainstream neo-classical doctrine of economics. He realised the laws of economics can explain a small part of human behaviour only. The logic behind economic and non-economic actions is essentially different. The mainstream players of modern economics solved the issue of the complexity of actions by merging economics with the intended choice function of all the other areas. It is exactly the opposite of what Pareto thought to be the objective of economics. The theory of rational choice has risen to become a general language used to explain any version of human interactions from economics to politics, or from culture to religion. The following opinion illustrates the above:

> *"The present methodology of economics (and physics) is characterised by imperialistic tendencies: they repeatedly aspire to account for almost everything"* (Cartwright, 1999:1).

Such methodology was the basis of the assumption of individual behaviour that, most of the time, *individuals are driven by their pursuit of rational calculation* to ascertain their interests. In contrast, as emphasised by Bruni (2010) – following

Pareto – "most of the time" *passions, sensations and ideals* drive individual action, although the latter cannot be analysed with the "logic" of economics.

For Pareto, the problem of logical individual actions led to his attitude to psychology: *he wanted to eliminate a component of metaphysical origins from economics.* The problem was how to verify the assumption of the existence of neutrality curves when one needed to move towards experimental facts. For Pareto, founding economics on psychology or on the facts of choice was the epistemological question of trust. Pareto believed the analysis of indifference curves was a safe basis while a psychological analysis was *"not scientific"*, as it was less certain and always required self-observation (Moscati, 2023).

At the beginning of the 20th century, the question of how to differentiate science from "non-science" was a demarcation line in epistemology. It was also fundamental for Pareto's theory. As reiterated by Bruni (2010), it was of key importance for Pareto to *distinguish science from metaphysics.* Thus, he opted for a rational choice in economics, he believed it was based on objective facts, and *he rejected all subjective and psychological dimensions as basic components of preferences.* For Pareto, the experimental (empirical) dimension is above all in objective verification: external experimental data were the primary scientific elements for him.

Pareto regarded psychology to be an experimental discipline. He envisaged the time might come when science develops so that one could set out from the facts of psychology and could use psychology as the basis of their theory, but – from the aspect of epistemology – he thought psychology was more fragile than the "facts of choice".

With attention to Fechner's and Wundt's experimental psychology, Pareto (1909) wrote the following in his „Manual":

> *"In the great variety of economic uses, there are many that are too far removed from the phenomena to which Fechner's law applies. It is better to resort directly to experience, and the latter shows us that for a great many users or consumptions the elementary utility does indeed diminish in the quantities consumed"* (Pareto, 1919:IV. 3).

Mostly influenced by neo-positivist philosophy, economics in the 1930s followed Pareto's experiment and experience-based theory. In terms of their methodological approach, the ideas of Hicks, Allen, Samuelson and others were directly linked to Pareto's (1909) "*Manual*".[9] It is particularly true with regard to rational choice

9 In the 1930s Hicks, Allen and Samuelson and others rediscovered Pareto's theory of choice. They presented Pareto as the person laying the foundations for a new economics, whose theory is based on facts and is free from metaphysical categories that cannot be observed, such as pleasure and usefulness.

in Pareto's antagonism to metaphysics and ordinalism. Edwards (1954) underlined that Hicks–Allen (1934) *tried to cleanse the theory of choice from its remaining elements of self-perception.* They adopted the conventional view of economics related to problems of indifference without referring to the concept of ordinal utility (however, the concept of the ordinal scale of preferences was included in their origination of indifference curves).

Moscati (2013) concluded that as a result of the so termed ordinalist revolution triggered by Pareto, theorists dealing with utility influenced by Hicks's (1939) "Capital and Value" *distanced themselves from cardinalism* and opted for the ordinal approach to utility.[10] The theory based on ordinal utility indices includes the *rejection of earlier concepts of* the utility theory, which are not ready for positive transformation by themselves. The concepts include the principle of reducing marginal utility, the definition of complementarity and the replacement of goods according to how a change in the marginal utility of one item of goods can modify the quantity of another item.

Hicks–Allen (1934) confirmed Pareto's opinion that the measurability of utility had become superfluous. In fact, Hicks and Allen did not only eliminate reducing marginal utility but also utility itself. They attempted to build a theory that relied on the *perceivable behaviour of choice* alone, and which could be described with indifference curves. The *marginal rate of substitution* had become the core of their analysis, which meant a quantitative and observable entity independent of utility.

Hicks (1939) and Samuelson's (1947) works (already completed in 1940) reflected the significant intellectual impact of the ordinal juncture, the canonisation of the subjective value theory and the elimination of utility indicating the dominance of the ordinal concept.

Hicks–Allen's (1934) work quoted repeatedly here, so to say predicted a major transformation of the content elements of the theory of choice when they wrote the following:

> *"The methodological implications of (the new) conception of utility (…) are far-reaching indeed. By transforming the subjective theory of value into general logic of choice, they extend its applicability over wide fields of human conduct"* (Hicks–Allen, 1934:45).

The same idea is expressed in *"Value and Capital"* by Hicks (1939):

10 At the end of the 1930s a series of studies by Samuelson (1937/1938a/1938b/1938c) heralded an extreme ordinalist juncture by creating the category of "declared preference" often referred to later.

> *"There are a great many extensions appearing at once when we consider how wide is the variety of human choices which can be fitted into the framework of the Paretian scale of preference"* (Hicks, 1939:24).

4 RISE OF THE PREFERENCE-BASED THEORY OF CHOICE

The ordinal juncture had two major consequences. One is a move towards a preference-based approach and the other is attention directed onto the ordinal utility function. The two were prioritised inasmuch as preferences as the principle of economic analysis were ranked higher while the utility function was a simple useful tool to represent preferences numerically. On the other hand, as Moscati (2023) emphasised, individuals are capable to perform much more complex assessment of preferences than simply listing the options.

An important moment in focusing on preferences was when Ramsey (1926) proposed to isolate loans from preferences as the grounds for choice under uncertainty while maintaining subjective probabilities at the same time. Moscati (2023) thought it was significant that the concept of preferences had several advantages for ordinalists. Diversity and criticism have characterised the interpretation of the part played by utility for a long time. According to the conventional concept, utility is connected to satisfying one's needs, achieving one's wishes or it causes pleasure, drives egotism or generosity. For ordinalists, ranking preferences may express any types of behaviour: material, egotistic, altruistic or masochistic, healthy or unhealthy, moral or immoral.

As another consequence, preference renders learning about economic implications of psychological considerations less important. According to the ordinalist approach, only the final preferences are relevant in economic analysis. Still, the complex psychological processes generating them are important for psychologists while they are not completely irrelevant for theorists of economics, either.

The third reason why ordinalists love preferences is that, although preference is a mental concept, it can be directly observed and is directly linked to the behaviour of choice unlike utility that cannot be directly observed. While the relationship between utility and choice is indirect in the sense it moves along the process of maximising utility, the relationship between preferences and choice is direct.

Beginning from the 1940s, a radical change occurred in the theory of rational choice both in economics and psychology. Objective rather than subjective probability came to be preferred and the model of probability of frequency was applied. Ordinal utility was replaced by cardinal utility, the earlier exclusive importance of utility was turned into the dominance of preferences.

The axioms of the theory by von Neumann and Morgenstern (1944) refer to the individual preferences of choosing between risky alternatives and involve a utility function that is suitable, by itself, for linear transformation. Although von Neumann and Morgenstern termed this specific form of utility "numerical" and not cardinal utility, they (hereinafter: NM) further developing Pareto's (1909) opinion, stated the term was related to ranking utility differences. NM termed their own approach as numerical because it was a safer and simpler solution compared to the approach originally proposed by Pareto, which emerged in the discussions of the 1930s:

> *"Our procedure, as distinguished by Pareto's is not open to the objections based on the necessity of artificial assumptions and a loss of simplicity"* (von Neumann–Morgenstern, 1944:29, footnote 4).

Edwards (1954) presented the most convincing interpretation on the depths of NM's model from the aspect of the theory of choice. He said NM was striving to provide a mathematical analysis of a wide range of problems of choice, particularly the ones *laying the foundations of strategic decisions*.[11] NM set out from the idea that *game theory* wants to find one or several solutions for any game. According to his theory, there is just one thing defining gaming behaviour: the amount of money won or lost, or the amount expected based on chance events. NM believed the expectation involved in the game was irrelevant and added:

> *"We have (…) assumed that (utility) is numerical (…) substitutable and unrestrictedly transferable between the various players"* (NM, 1944:94).

Heukelom (2007) calls attention that in NM's book the game theory is an extension of the decision theory to situations in which *rational choice depends on other players' behaviour.* Placing the application of the utility theory into the context of game theory is one of the most important novelties of the NM model. NM recognised that DB's utility function could only manage and explain the problems of aleatorical games but not those in interpersonal situations. DB's concave utility function only explains that individuals are willing to spend money to reduce risk. However, it is assumed that individuals will not commit themselves either for a fair or for a less fair game. *A real situation to decide, however, is a problem in the theory and practice of economics that goes beyond the cases of security and aleatorical games.*

11 Edwards (1954) explained the connections between game theory and the theory of rational choice. Accordingly, game theory provides practical advice on how to play in a complex game; game theory at the same time is a crucial point in the strategy of choice that can minimise maximum loss of money.

Von Neumann–Morgenstern's (1944) epoch-making work was key in the development of decision-making under risk. They have found an „economic man" may prefer one state over another or may be indifferent in their choice. In their view, decision theory can mainly be considered to be a prediction.

Theorists of economic decision-making worked for a long time to develop formulas on how individuals choose one of the possible alternatives. They usually focused on utility, i.e., alternatives a decision maker must choose from. They assumed people behaved rationally, i.e., had transitive preferences and maximised utility with their choices. Compared to the traditional utility-maximising theory of risk-free choice, NM's idea according to which indifference curves allow risk-free choice *without the exact measurability of utility* at any scale was a challenge.

NM's (1944) work created an axiomatic version of the preference-based expected utility theory. They pointed out that provided a decision-maker's preferences observe certain axioms – in the case of choice under risk -, individuals will choose the risk option which is linked to the highest value expectation of the cardinal utility function $U(\tilde{x})$n.[12]

In the middle of the 1940s, NM's concept of expected utility brought about a sharp turn in the theory of rational choice. Preference had become the cornerstone of the theory; the authors assumed individuals could compare two lotteries – L_1 and L_2 – to decide which of them to prefer and to compare with the risk-free preference of a certainly available group of goods.

While Pareto (1909) and the ordinalists adopted the category of preference and thought it corresponded to common sense, but found it vaguely defined, von Neumann–Morgenstern (1944) regarded it to be a core concept of economics. According to Edwards's (1954) important evaluation, the five axioms of NM's procedure – completeness, transitivity, continuity, reduction and independence – are *of an ordinal nature* in the sense they only present a ranking of the options. Therefore, the specialty of NM's version of expected utility *is not its axiomatic character but that the axioms are preferences rather than utility functions.*

Moscati (2023) was right to say NM's theory fails to explain why a decision-maker prefers one lottery game over another. So, the NM model cannot state a decision-maker prefers one lottery over another because the marginal utility of money is reducing, or because he is risk averse or because he is afraid of losing his money.

12 The sign '~' marking the function $U(\tilde{X})$ is what differentiates von Neumann–Morgenstern's utility function from Daniel Bernoulli's utility function $u(x)$.

Therefore, in NM's theory *the decision-maker's preference of lottery games is a basic factor of the analysis, which lacks an explanation*.[13]

The position of NM's expected utility is different from the procedure of "multiply and add" used in DB's expected utility. In the latter, the decision-maker is assumed to multiply the utility of each outcome with the relevant probabilities and then to sum up the values $u(x_i)p_i$ to calculate the expected utility of the lottery $\sum u(x_i)p_i$ and thus to select the lottery with the highest utility $\sum u(x_i)p_i$ after comparing the different versions.

Compared to the DB model, the decision-making mechanism is different but simpler in NM's preference-based expected utility version. Based on complete and transitive preferences, the decision-maker *will rank* all lottery games and select the most preferred one. There is no need for calculations. The fundamental reason of why utility functions $U(\tilde{x})$ and $u(x)$ are not equivalent with each other is that the function $U(\tilde{x})$ can be derived from the decision-maker's preferences of the lottery games , thus, it *reflects and combines all potential factors* impacting the preferences relating to uncertain options. Moscati (2023) has revealed the key significance of NM's theory is also the main problem of the model, because one *cannot reveal and identify the divergent factors* within the model of expected utility, which could affect the decision-makers' preferences of the lottery games.

Moscati (2023) raises the question whether NM's preferences have anything to do with psychology. He thinks the preferences are not rooted in common sense or everyday psychological considerations, but they are formalised binary relations meeting the criteria of necessary features, such as completeness, transitivity, and stability in terms of both the domains of choice and the hypothesis of the methods of finding. Moscati, however, doubts if all the transitive and stable preferences appear *in the minds* of decision-makers.

Therefore, one can prove that both NM's expected utility risk preferences and the preference-based decision-making mechanism should be interpreted as the utility function $U(\tilde{x})$, i.e., it must be regarded to be *a theoretical construct* used by theorists of choice to attempt to predict the *behaviour of decision-makers under risk,* while psychology cannot be involved in the minds of decision-makers.

13 According to Moscati (2023), Neumann applied the axiomatic approach in several areas in the 1920s and 1930s, such as in mathematics, logic, geometry and physics. At the same time, Morgenstern studied the paradoxes used to find an answer to the question how the interactions of two players depend on the prediction of their actions considering the other player's action (von Neumann, 1928)

5 SIMILARITIES AND DIFFERENCES OF BASIC MODELS OF CHOICE

David Bernoulli (1738/1954) stated the monetary value of an option could not be the right measure for choice. In the 1940s, von Neumann–Morgenstern *rejected psychophysicist bases* and built individuals' principle of decision-making *on the objective amount of money.*

NM (1944) triggered a major change in the theory of choice by assuming – through a theoretical simplification - that utility is equivalent to the amount of money in question:

> *"We wish to concentrate on one problem – which is not that of measurement of utilities and of preferences – and we shall therefore attempt to simplify all other characteristics as far as reasonable possible. We shall therefore assume that the aim of all participants in the economic system, consumers as well as entrepreneurs, is money, or equivalently a single monetary commodity. This is supposed to be unrealistically divisible and substitutable, freely transferable and identical, even in the quantitative sense, with whatever 'satisfaction' or 'utility' as described by each participant"* (von Neumann–Morgenstern, 1944:8).

NM (1944) clearly rejected DB's ideas, and re-defined utility in the sense of efforts to make money. To assess the weight of the change, one can say NM turned back time to the age preceding DB when rational choice depended on money of absolute measure and objectivity. In NM's (1944) theory individuals wanted to maximise the monetary benefit of the game and wealth rather than utility in the sense of DB. Players in the NM model maximised money or the equivalent of money but termed that money utility. Therefore, one can say the value used for calculations was objective at the time preceding DB, it became subjective at the time following DB and again became objective at the time following NM.

Maximising expected utility is another key change in NM's model inasmuch as the earlier dominance of ordinal utility was replaced by cardinal utility re-discovered. Baumol (1958) calls attention the term "cardinal" has two meanings: one is an inward looking absolute marginal *measurement of pleasure* in psychophysics regarded to be rooted in metaphysics, while the other is a term of the game theory where the term "cardinal" is fully operational in nature.[14] Baumol underlines 'measuring the strength of a sensation' is irrelevant in game theory which is the foundation of NM's utility theory. The measure of cardinal utility has been set up

14 Baumol (1958) regarded an interpretation of the NM index as a naïve effort if reference is made to cardinal value in its old sense.

for calculation purposes and out of the wish for prediction so that builders of theories could decide which of many risky variants should be preferred by a player.

NM's utility measure is *cardinal* in the sense it provides richer information than what can be gained from a standard series, for instance, by asking individuals about their preferences. In NM's model cardinal utility is an economic solution that only requires asking about one individual's preferences by ranking the prizes of the lottery games. According to Baumol (1958), NM's utility index, provided it can be applied to a given individual, allows the interviewer to make assumptions on the rankings of all the other alternatives made by the same individual relying on the answers received earlier. Still, one must emphasise the *NM index provides information about an individual's ranking of preferences only.*

The prediction of behaviour related to the choice between two lottery games can best be made using numerical calculations. In NM's model the choice of a given game can be predicted from how the individual ranked the prizes to be gained. You need to define a random cardinal utility value that allows making the prediction.

In the 1930s, several authors for instance, Allen (1935), Alt (1936), and Samuelson (1938c) examined in detail the possibility of gaining cardinal utility by assuming that individuals are capable of ranking the differences of utility of risk-free alternatives. However, theoreticians dealing with utility remained sceptical about the option, since rankings of utility differences have no clearly observable equivalent in the sense of actions of choice, so, they *are simply based on self-observation, which cannot be considered credible proof for the evidence* (Allen, 1935).

Through a critical analysis of NM's model, Moscati (2023) called attention to a major contradiction in the theory of choice based on utility. He set out from the fact that there is direct connection between the gradient of the utility function u(x) and decision-maker's risk attitude. The decision-maker is actuarily risk averse (risk seeker) if and only if the function u(x) is concave (convex). However, by NM's utility concept the correspondence *does not indicate a causal relationship.* It is illustrated if you compare DB's and NM's models (Moscati, 2023:34).

> *„Since the utility function u(x) is concave, the individual is risk averse",*
>
> *„If the decision maker has risk averse preferences, the utility function* $U(\tilde{x})$ *aimed to register them will be concave".*

This is where the contradiction appears. If preferences are *unstable in time* in terms of the domains of choice or the methods of finding, the completeness and transitivity axioms of the preferences affecting risk variants *are limited in capability to explain decision-makers' choices.* Another form of preference instability is connected across domains to the possibility that a decision-maker's *risk attitude* changes when they face a specific type of options.

According to Moscati (2023), the core problem of NM's model is that one cannot reveal the divergent factors within the theory of expected utility which can affect a decision-maker's preferences regarding the lottery. The issue is manifest in the fact that the model of expected utility only has one "free" variable, namely the function $U(\tilde{x})$. Therefore, $U(\tilde{x})$ will definitely be a *black box combining and reflecting all psychological factors* that affect the preferences of a decision-maker. Other models of decision-making, than the expected utility models of decision-making – for instance Kahneman–Tversky's (1979) prospect theory – is trying to *open up* this black box and untangle those factors by attaching additional free variables to the model of expected utility.

Works by Kahneman and Tversky significantly contributed to the theory of choice in economics. The authors have been analysing the standard assumptions regarding the stability of preferences and the conditions of their invariance with respect to different variants of risky outlooks since the beginning of the 1960s. The following quote illustrates the importance of the results of their research:

> *"[they] refuted the claim that von Neumann–Morgenstern theory system is a good predictor of how ordinary people behave in making decisions"* (Binmore, 2000:58).

In their first thesis, Kahneman and Tversky (1979) (hereinafter: KT) stated the following. In line with the psychophysics of value, *individuals are risk averse with respect to benefit, but they are risk-takers in the domain of loss.* According to the psychophysics of value, individuals overweigh certain things and improbable events, but they underweigh moderately probable occurrences. Decision problems can be described or framed in several ways, which will trigger different preferences in contrary to the invariance criterion of rational choice.

In KT's approach, the so termed prospect theory, NM's utility concept is replaced with the *psychological value of* benefit and loss.[15] Kahneman–Thaler (1991) believes NM's objective decision utility has been proved not to be a good basis for the academic description of rational decision-making. DB argued the basis of calculations should be the subjective value of financial loss or benefit. According to Kahneman–Thaler (1991), subjective "utility experienced" only can be a good basis for a descriptive theory.[16] A theory at a distance from maximising expected

15 This psychological value is similar to Edgeworth's (1911) concept of utility, as pleasure experienced, that can be measured objectively with the help of the technical solution termed "hediometer" (Baccini, 2011).

16 To differentiate the normative and the descriptive theory in terms of their usefulness, Kahneman (1994) identified conceptual difference between "decision utility" and "experienced usefulness". Decision utility refers to the standard application of utility in the theory of choice and can be regarded to be identical to NM's concept of utility. It is the objective monetary value of an option,

utility has been elaborated in the field of behavioural economics. While the theory of expected utility is a valuable tool to analyse *how rational individuals make decisions* under uncertainty, their behaviour observed *does not support that.* KT (1974) was the first who provided proof that *the theory of expected utility could not offer a complete picture of how individuals actually make decisions under uncertainty.* The authors led experiments demonstrating divergence from the theory of expected utility: it has turned out *individual behaviour violates,* under certain conditions, the axioms of rational choice of the theory of expected utility.

In KT's theory, NM's function of expected utility has been replaced with the *value function* $v(y)$ including two slices of benefit and loss. KT's value function is analogous to DB's utility function of $u(x)$. It expresses the subjective evaluation a decision maker connects to risk-free outcomes, which is of a cardinal nature. As Moscati (2023) refers to it, there are some features differentiating value functions from the utility function $u(x)$. For simplicity's sake, let us take an example to illustrate the features where the outcome is an amount of money.

Firstly, according to KT (1979: 277-288), the principle of perception and judgement means that individuals perceive different outcomes either as benefit or loss compared to *some kind of reference point,* which differs from the model of expected utility, in which ultimate riches is the basic criterion.

Secondly, KT (1979: 279) believes psychological proofs exist showing that *the threat of loss is stronger than the benefit outlook of the same amount.* That is termed *abstinence from loss.* The latter means that the (slice of) the value function of loss is *steeper* than the curve of the value function of benefit – the difference can even be double.

Thirdly, for KT, the marginal subjective value of both the benefit and loss decreases as their size increases. Accordingly, the value function' slice $v(y)$ is concave, similarly to DB's utility function $u(x)$. In terms of loss, the assumption of reducing marginal utility means the value function's slice $v(y)$ is convex.

The original version of the prospect theory has the same $\sum u(y_i)p_i$ structure of expected utility – multiplication and addition as DB's model of expected utility. In both cases, an index represents the subjective value attached by the decision maker as weighting – via multiplication – multiplied by the parameter representing actual or perceived probability linked to the event the outcome originates from. Next, the weighted subjective values of the outcomes are added, and the resulting numerical value is taken to express *the subjective value* presumed by the decision maker for the whole game.

or an option expressed in money terms. Subjectively perceived usefulness is called experienced usefulness.

Ramsey's (1926) idea triggered a train of thought as a result of which a decision maker's beliefs can be indirectly understood from the preferences or choices rather than directly from self-observation. The key issue of the preference-based approach – of how to find out the belief – is that a decision maker's preferences of choosing one of the alternative actions *do not only depend on the beliefs affecting the probability of events but also on other factors, such as preferences relating to the outcomes, risk attitude and the pleasure or pain that can be associated with the actions during the game.* That is why Moscati (2023) believes it important to neutralise those additional and important factors when one focuses on finding out the beliefs from preferences and selective behaviour.

It is clear from this sub-chapter that the original version of KT's prospect theory can best be interpreted as an extension of DB's utility-based expected utility rather than NM's preference-based expected utility.[17]

6 SHARP TURN IN TREATMENT OF PROBABILITY AND UTILITY IN THE THEORY OF CHOICE

When NM's (1944) model of maximising expected utility was established, *there was a sharp turn* in two basic concepts. *Subjective probability,* which had been widely accepted, was replaced by *objective probability* in NM's theory. The other change affected *the ordinal juncture,* which had been dominant, as cardinal utility was rediscovered and rehabilitated. The change was in sharp contrast to lines of thinking in the earlier decades.

Ramsey (1926) was the first to set out a decisive opinion in the theory of choice of economic decision making. He stated probability *was only linked to the given individual.* He said probability was limited by personal belief and not built-in knowledge; therefore, probability is subjective in decision making.

17 It is an interesting momentum in the development of the theory of choice in economics when theorists do not believe NM's model of expected utility and the utility analysis based on psychophysics are mutually exclusive. Psychology was definitely limited in NM's theory of utility. Luce (1989) indicated the problems of psychophysical measurements, Fechner–Weber's concept, focusing on the comprehensible difference of sensations. Luce wrote the following: „*I was excited by von Neumann-Morgenstern's concept of expected utility together with Weber-Fechner's problems of psychophysical measurements, and any possible connection between the two*" (Luce, 1989:249). In an earlier work, Luce (1956) tried to include psychophysical considerations in the utility analysis. In addition to economic considerations, Luce–Suppes (1965) used the methods of psychology and philosophy in their utility analysis, which indicates a connection of economic thinking with the cognitive sciences.

Earlier at the end of the 19th century, Edgeworth (1884) described probability as something with an irreducible dual nature. He referred to two different spheres of probability: one is objective connected to the observed frequency of certain events, while the other is subjective, a mental condition linked to their frequency.

If probability can be described as partial or incomplete belief accepted, the subjective state of consciousness can be regarded to be a belief. The Oxford English Dictionary offers the following definition of 'belief':

> *"The belief is defined as mental acceptance of a proposition, statement or fact, as true, on the grounds of authority or evidence; assent of the mind involved in this assent"* (Oxford English Dictionary, 1986:vol 2, 86).

The definition regards belief as something accepting a statement as true *without regard to its objective validity.*

As opposed to the acceptance of the theory of choice based on subjective probability, R. Mises (1928) re-interpreted probability based on relative frequency. In his proposal, probabilities are linked to *series of events* rather than single events and the theory is an objective approach. In his new approach, "relative frequency" is the probability of an event in an experiment, i.e., the relative occurrence of an event is an infinite chain of identical experiments. The "relative frequency" of infinite repetitions could only be *idealisation,* since how could you examine the probability of events if they are inherently "unique" in situations of decision making.

Von Neumann–Morgenstern's (1944) choice of probability and the preference of cardinal utility were closely interrelated.

In their concept, an individual's utility function $U(\tilde{x})$ can be identified, so the utility index can be individually defined as its linear transformation, which is the cardinal measure of utility. As Moscati (2023) indicates: it takes place through the direct observation of an individual's choices at risk and allows to describe the different behavioural character of individuals at risk. NM derived individual values – for prioritisation – from the axiomatic treatment of numerical utilities. The construction applied *probability as the individual estimation of utility.* NM's starting point was that a subjective concept of probability would not serve their efforts. Thus, they argued for the following:

> *"We argue for a perfectly plausible alternative interpretation of probability, which is the long-run frequency. This directly supplied the necessary numerical starting point"* (NM, 1944:19).

Savage (1954) added an interesting comment on NM's choice of probability:

> *"Objective probability can only be fruitfully applied to repeated events but cannot be used to produce a version that is the most promising of several options of action since probability cannot be related to the truth of the statements"* (Savage, 1954:4).

Moscati (2014) underlines the concept of *cardinal utility was marginalised* in the 1930s, since at the time of the ordinal juncture, the thinkers dealing with utility supported a strictly ordinal approach to the utility analysis. However, NM's objective was to define the concept of utility with no regard to earlier efforts regarding utility ranking. Their main interest was that cardinality should not rely – more or less – on the measure of "pleasure" or "satisfaction" from goods based on self-observation. They attempted to separate their approach from cardinality based on the comparability of the differences of preference.

Fishburn (1989:131) provides an interpretation of the cardinality of NM's model of expected utility. Accordingly, the basic feature of deriving a cardinal utility index is the analysis of a situation where an individual *can choose a safe outcome and two other outcomes, whose probability is given, and random figures are attached to the two outcomes in accordance with the individual's preference ranking.* This proves NM's discussion of utility was mainly opportunistic.

Because the axioms of the theory of expected utility implied the existence of the cardinal utility function, i.e., a function that is ready, in itself, for linearly increasing transformation, the rise of the theory of expected utility in the 1940s was linked to the *rehabilitation of the concept of cardinal utility.*

Following NM's (1944) model, the contribution of theorists dealing with utility promoted shaping a new concept of utility evaluation, which confirmed the status of the rehabilitated cardinal utility (for instance, Friedman-Savage,1952; Stortz, 1953; Ellsberg, 1954). Moscati (2014) emphasised that, in the new concept, *cardinal utility is not the opposite of ordinal utility, but it is its alternative, and it is not compatible with utility "based on self-observation", either.* Instead, ordinal and cardinal utility can be understood as two identical modes of measuring utility, i.e., the numerical values must be fitted to the objects of choice so that it should be suitable to predict selection behaviour while - provided the theory of expected utility is accepted – the numerical values of cardinal choice are suitable to predict choice at risk.

7 ACADEMIC PRECISION OR OPERATIONAL EXPEDIENCY: SYNTHESIS IN THE THEORY OF CHOICE

Following the publication of von Neumann–Morgenstern's (1944) model of expected utility, at the turn of the 1940s and 1950s, the theorists of economic decision theory contemplated on the content of NM's utility index and the applicability of such a special version of utility in decisions. NM's model was built on new approaches including a *dominant preference base, the replacement of subjective with objective probability or ordinal with cardinal utility.* Two major theoretical

novelties supported the operational consolidation of NM's utility. One was Savage's (1954) model of *subjective expected utility,* the other being Friedman's (1953) „as if" methodology as it had become accepted by decision theorists. Savage's (1954) findings established a synthesis by unifying the axiomatic treatment of subjective probability and utility. Friedman's (1953) interpretation of the theory of choice promoted the *predictability of decision-making behaviour.*

Ramsey (1931), de Finetti (1931) and Savage (1954) shared reservations about whether individuals' beliefs in decisions can be revealed through self-observation or interviews. They thought the identification of beliefs was more complex. What is more, Savage (1954:27) was sceptical regarding their validity. Following along the line of thought of his predecessors, Savage (1954) created an axiomatic model that included both subjective probability and utility and related to uncertainty rather than risk. Relying mainly on Ramsey's (1931) idea, Savage (1954) proved the theory of expected utility can be extended if the probability of uncertain outcomes is not defined objectively, which expresses a decision maker's subjective belief in terms of the probability of outcomes. According to Moscati (2014), that extension of subjective probability confirms the status of the theory of expected utility as the model of choice at risk.

Feduzi et al. (2014) underline that both Ramsey (1931) and de Finetti (1931/1937) were pioneers in connecting probability analysis with the theory of choice in economics. Savage (1954) expanded the connection. He showed that, provided decision makers wanted to meet the requirements of rational choice, it would be identical to choosing from outcomes in such a way that coincided with their expectations of their own utility values. Their expectations reflect that a decision maker considers some well-defined value-pairs of probability-utility. The expectation is generated so that the decision maker considers some well-defined value pairs of probability-utility. Savage's most significant contribution is he analysed the two components at the same time. *Savage's (1954) subjective probabilities are used to model situations where the objective probability of the outcomes is unknown.* However, Finetti–Savage (1962:82–84) state when using the subjective approach as probability that it is "not realistic" and "not practical" to restrict probability to situations where either frequency or symmetry are present. Instead, the subjectivist view assumes that in case *probability must be wide, complex and uncertain, relying on fragmented and incomplete information,* it [probability] is the "arbitrary opinion" of the individual wording their judgement.

Savage's work (1954) can be regarded to be the synthesis of the latest results of the theory of choice, because he used a preference-based approach while building subjective probabilities into his model. Thus, he extended NM's theory to cover cases of expected utility where no objective probabilities are available. Savage's results (1954) confirmed NM's construction, and thus *the model of subjective ex-*

pected utility has become the main paradigm of decision making. According to it, maximisation of expected utility based on subjective probability is the rational route of the decision-makers' behaviour, which – under the conditions of uncertainty – is suitable to manage the problems of economic decision making in a convincing manner. Savage (1954) maintained that the theory of choice is about rational decision-making behaviour; it is not an a priori gift to be thought of as a normative theory. Accordingly, *the theory of choice can be regarded to be a prediction* of how individuals will behave in decision-making situations, at the same time, it must be treated as a logical criterion of consistence in decision-making.

Setting out from a primarily psychological perspective, Edwards (1954) raised the question about NM's (1944) and Savage's (1954) model whether they can be used to consider practical decisions under risk. Edwards (1954) also developed conditions for their application. Firstly, he spoke about developing a *suitable scale of the utility of money, and the estimation of subjective probability.* The criterion of the suitability of those scales must be how successfully they can predict choice rather than *predicting what the choice has originated from.* A choice can really be observable if predictions relate to a wide circle of situations of choice. Unlike the subjective scales of psychophysics, they will be quite different from one individual to the next, so one must prepare a new determination of each scale, separately for each person. One can only hope the scales *are not going to change much in time.*

With respect to Savage's model of subjective expected utility, Edwards (1961) says he has found through his psychological experiments that individuals mostly behave in line with the normative theory. An exception, where they diverge from the normative theory, is when they come across new information about uncertain events.

A critical aspect of NM's (1944) theory of expected utility is *the inclusion of cardinal utility in the model.* At the beginning of the 20th century and again in the 1930s, the followers of the ordinal juncture considered ordinal utility to be more realistic, since *it is difficult to measure utility numerically.* The idea of cardinal utility has proved to be useful for the theory of rational choice. Acceptance of NM's expected utility did not mean acceptance of the ordinal approach or a return to a pre-Pareto utility concept. NM's expected utility theory did not only provide academic verification of the use of cardinal utility, but also proposed a practical method for its experimental measuring.[18]

In addition to promoting the idea of cardinal utility in order to lay the foundations of the theory of rational choice, Baumol (1958) also emphasised the cardinal

18 Nostel–Nogee (1951) presented the first experiment where the (cardinal) utility of money was measured for fifteen individuals based on their choice between monetary games.

utility applied in NM's model was not identical with the category termed the same *from the times preceding Pareto.* NM has shown how to look for numerical values that can help predict an individual's ranking of the lottery game. In fact, NM's axioms represent ordinal measures. The solution, in effect, measures the cardinal utility of the game, which is not cardinal utility *in the old sense.*

Moscati (2023) goes into detail to describe the consequences of the academic-consolidation steps of the time following the publication of NM's expected utility (Friedman-Savage, 1948; Friedman-Savage, 1952; Friedman, 1952). In general terms, the discourse about NM's (1944) utility function $U(\tilde{x})$ attempted to *re-conceptionalise* measuring utility and the concepts of ordinal-cardinal utility. According to his novel *operational* view, measuring utility consists of the figures attached to its outcomes on the one hand, and on the other hand, the lottery games or their outcomes make up a definite set of operations. Attaching numerical values to occurrences is mostly random and conventional. The essential restriction is that the *numerical values attached must allow decision-makers to predict an individual's behaviour of choice.*

Moscati (2023) believed the decisive change in NM's operational interpretation of expected utility was that *the contrast between ordinal and cardinal utility faded* as opposed to measuring utility as proposed by Friedman (1953) in his prediction-oriented solution. Thus, it cannot happen that utility is inherently cardinal or ordinal. Instead, there are ways of the same weight in the attachment of ordinal or cardinal utility. Accordingly, ordinal utility is a suitable method to analyse market equilibrium, while cardinal utility should be applied in other areas of economic analysis, for instance, the theory of choice at risk or temporal decisions.

Earlier, cardinal and ordinal utility were mutually exclusive alternatives in the theory of choice. However, *a peaceful coexistence has been present* since the beginning of the 1950s. It is, surely, thanks to the approach commenced by Friedman-Savage (1945) and refined by Friedman (1953), i.e., the so termed "as if" interpretation of expected utility. In his latter work Friedman stated the relevant criterion to judge the academic value of economics is not how realistic an approach is but how much it is capable to make accurate predictions. If it is related to the predictions of decision-makers' behaviour, new vistas open up to solve the dilemmas of the theory of choice.

Friedman (1953) spoke about the importance of prediction as follows:

> *"The relevant question to ask the 'assumptions' of a theory is not whether they are descriptively 'realistic', for they never are sufficiently good approximations for the purpose in hand. And this question can be answered only by seeing whether theory works, which means whether it yields sufficiently accurate predictions"* (Friedman, 1953:15).

On the other hand, if the predictions of a model are proved to be unrealistic, but the theory makes accurate predictions for a certain category of economic events, one can say decision-making works *as if* the assumptions of the concept were realistic.

Moscati (2023) emphasises if you apply the "as if" hypothesis for expected utility and other models of decision-making, it means *those models must reckon with the individuals' observable choices without pretending they have learnt the psychological mechanisms generating those choices and providing their grounds.*

Nevertheless, when a decision-maker applies the "as if" approach, they are uncertain whether the mechanism actually works *in a decision-maker's mind*. The theorists of choice may believe, and they clearly acknowledge that a positioned mechanism and its components *can only be regarded to be fictional structures.*

The theorists of choice have a peculiar approach to the "as if" solution assisting the operationalism of NM's model. Most of them accept the theory of maximising expected utility and they identify, in thought, with the function embodying good behaviour represents in the selection of lottery games (Luce–Raiffa, 1957:31–32). Preference allows decision-makers to rank the lottery games available and select the most preferred one. By contrast – according to Binmore, 2019:19–20 – they regard NM's utility function *to be a theoretical construct,* which allows to explain, describe or predict decision-makers' behaviour at risk, but they do not accept there is any connection in the decision-maker's mind.

The issue of equivalence or difference between DB's and NM's utility functions, i.e., the relationship between the functions $u(x)$ and $U(\tilde{x})$ has been a long-term dilemma of the role played by expected utility in decisions. If equivalence is assumed, *the ordinal utility approach cannot be rejected* with cardinal utility having an exclusive player. The difference between NM's approach and the ordinalists is the latter took it for granted that preferences could be represented numerically with the help of utility functions. In contrast, NM identified the features – axioms – corresponding to the binary relation "greater than or equal" in the set of lottery games which must not only guarantee numerical representation but must also ensure the representation has the form of expected utility (cf.: Moscati, 2023).

The validity of Friedman's (1953) efforts for operationalism is supported by the fact that individuals can behave in an "as if" manner when they compare the expected utility of lottery games, or in the event of NM's preference-based expected utility, where people behave "as if" with stable risk preferences observing the five axioms of expected utility. *If you acknowledge the five axioms of expected utility are ordinal by their nature, the cardinal utility $U(\tilde{x})$ cannot be interpreted to be equivalent with a utility function representing decision-makers' preferences.* Risk

options on one side and NM's model of expected utility on the other side, since the latter does not include the rejection of the ordinal approach.

Friedman's „as if" methodology, specifically, the "as if" interpretation of expected utility is fully accepted by mainstream neoclassical theorists of economics with a slight distinction: they identify with the acceptance of risk preferences, but they voice their reservations about NM's utility function $U(\tilde{x})$.

Studies offering alternative models of choice in contrast to expected utility often criticise von Neumann–Morgenstern–Friedman–Savage's approach, because the latter and their followers defend expected utility from the suspicion it lacks psychological reality. Kahneman–Tversky's (1979) original prospect theory is the most influential of the models of choice at risk and uncertainty. If you want to answer the question how theories of choice coming after expected utility will be able to overcome the problems of expected utility, the answer is by replacing utility with value, by offering a different approach to benefit and loss and by treating risk attitude differently.

8 SUMMARY

This paper is about comparing three peaks in the development of the theory of choice, i.e., the models generated by Daniel Bernoulli, von Neumann–Morgenstern and Kahneman–Tversky in a historical context from the aspect of decision making with particular attention to some theoretical feedback.

According to DB's (1738/1954) basic idea, individuals' choices are not based on values expressed in money, but on the psychological value of their outcomes, i.e., utility. Thus, the psychological value of an aleatoric game is not the weighted average of possible outcomes expressed in money but the value of the utility of outcomes multiplied by the probability of each outcome. DB used a concept he had introduced, namely expected utility or immoral expectation as he had termed it, to calculate how much an individual would be willing to pay to avoid risk. DB observed that most individuals are unwilling to take risks (the probability of the worst outcome), so, if they must choose between an aleatoric game and a sure amount equal to the expected value of the game, they will opt for the sure amount.

Later Fechner (1860) formulated the idea as a law of psychophysics, where he assumed a connection between psychological value (i.e. the utility of money) and the actual amount of money. In Fechner's view the psychological reaction triggered by the change in the amount of money is inversely proportional to the amount of money owned initially.

DB's thesis of maximising expected utility had remained a basic reference and paradigm for a long time. However, Marshall (1920) and other theorists of the topic rejected the maximisation of utility as an explanation for choice involving risk. The reason was that decreasing marginal utility turned out to be irrelevant if you wanted to explain decision game theory.

Although returning to the psychological roots revealed by Daniel Bernoulli and Fechner, KT criticised the treatment of the utility of wealth. DB's initial idea was individuals are more or less satisfied depending on the utility of their wealth. KT emphasised an individual's satisfaction is subject to *how much and in what direction their financial-monetary position has changed compared to a point of reference.* According to Kahneman, dependence on the point of reference can be observed in all areas of sensation and perception. Since DB's model does not include the concept of reference point, the theory of expected utility fails to reflect the obvious fact that what is good for one individual, can be bad for another. Kahneman criticises DB because DB's model does explain one individual's risk aversion, but it cannot explain another individual's risk acceptance, although many people are willing to take a risk if they can only choose from bad options. (Kahneman, 2013:316–317).

Before NM (1944) published his epoch-making work, theorists of choice believed in line with a widely accepted view that *assuming decreasing marginal utility* was unnecessary to explain risk-free choice. They rejected maximisation of expected utility as an unrealistic assumption. NM's work has basically transformed the theory of choice as it challenged the rejection of the maximisation of expected utility. NM was aware utility cannot be directly measured, but there are options to obtain indirect measures. He said:

> *"under conditions based on the indifference curve analysis, the extra small effort is required to achieve numerical utility, where the expected value of utility is maximised by the choice of alternatives including risk"* (von Neumann–Morgenstern, 1944:17).

The hypothesis of expected utility revived by NM relies on the idea that DB's model could only treat insurance and aleatoric games effectively, but not the situations of interpersonal ones. Accordingly, NM's model of maximising utility could operate on objective probabilities, cardinal utility, and random probability rankings. NM's change of theory of high significance was made complete by returning to the thesis of "money value-utility" preceding DB.

Kahneman (2013) believed DB's model was weak because of an over-simplification of its theoretical grounds. He thought the point of reference was a missing variable in the model, i.e. the earlier state you could relate benefits and losses to. According to DB, it is sufficient to know the actual state of wealth to define its utility.

However, in KT's (1979) prospect theory, you should also know the state of reference. Thus, the prospect theory is a more sophisticated approach than the theory of utility. In it, benefits and losses are assessed compared to an indifferent point of reference termed the adaptation level. According to another consideration, the concept of decreasing sensitivity is valid in the same way in the dimensions of perception as in the evaluation of the changes in the amount of money owned. The most important attribute is the concept of loss aversion. Losses seem to be greater than gains in direct comparison or when weighed against each other. The value function of KT's prospect theory expresses psychological value (unlike DB's model, where the value of wealth carries it). KT has provided convincing proof that over-avoidance of risk related to minor losses cannot be explained by individuals' relationship to the different states of their wealth (Kahneman, 2013:325–326).

Despite their major differences, the basic models of the theory of choice also have similarities. On several occasions, they return to earlier concepts by accepting their approaches. To perfect the theory of choice, more academic and operational efforts are needed. The road is open for researchers...

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